Sequential Processes and the Shapes of Reaction-Time Distributions[1]

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Abstract

It is sometimes suggested that reaction-time (RT) distributions have the same shape across conditions or groups. In this note we show that this is highly unlikely if the RT is the sum of the stochastically independent durations of two or more stages (sequential processes) (a) that are influenced selectively by different factors, or (b) one of which is influenced selectively by some factor. We provide an example of substantial shape differences in RT data from a flash-detection experiment, data that have been shown to satisfy the requirements of type (a). Ignoring these requirements, we also note that in a large range of instances reviewed by Matzke & Wagenmakers (2009) in which the ex-Gaussian distribution was fitted to RT data from different conditions in the same experiment, most sets of distributions fail to satisfy a weak requirement for shape invariance.

It has sometimes been suggested that reaction-time (RT) distributions have the same shape across conditions or groups. Shape invariance of a set of distributions means that they differ by at most their means and time scales. Thus the distributions of X and Y have the same shape if and only if there are constants a and b such that $Y = a + bX$. For example, one proposal about the cognitive effects of aging is the General Slowing Hypothesis: With increasing age, all the operations of the central nervous system in most or all tasks become proportionally slower (Cerella, 1985; Eckert, 2011; Myerson et al., 2003a; Myerson et al., 2003b; Salthouse, 1996; Sleiman-Malkoun et al., 2013). In effect, with increasing age, time runs more slowly. Rouder et al. (2010) discuss other considerations that lead to shape invariance. Ratcliff and McKoon (2008) use the approximate linearity of Q-Q plots to argue that for diffusion model predictions and some data sets, the shapes of RT distributions are approximately invariant across experimental conditions and experiments (p. 895). And, according to Ratcliff and Smith (2010, p. 90), "Invariance of distribution shape is one of the most powerful constraints on models of RT distributions. . . . That the diffusion model predicts this invariance is a strong argument in support of its use in performing process decomposition of RT data."

The primary purpose of this note is to show that for a process organized in stages that have stochastically independent durations and are selectively influenced by experimental factors, it is highly unlikely that the distributions of RTs in several conditions in an experiment can have the same shape.

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Stage models

Stage models are ubiquitous in research on speeded tasks (e.g., King & Dehaene, 2014; Sanders, 1998; Schall, 2003; Schall et al., 2011; Sigman & Dehaene, 2008; Sternberg, 1998, 2001) and elsewhere (Borst & Anderson, 2015). For several sets of RT data, Roberts & Sternberg (1993) provide evidence for selectively influenced stages whose durations are stochastically independent. Even in Ratcliff's (1978) diffusion model, in which the decision process may be represented by multiple activations that grow in parallel, the decision stage is augmented by two additional stochastically independent processes arranged sequentially: an initial stage for stimulus encoding, and a final stage for response execution. In the application of the diffusion model considered by Gomez, Perea, & Ratcliff (2013), the duration of the encoding stage in a lexical-decision task is found to be selectively influenced by the relatedness of masked primes. In the experiments considered by Ratcliff & Smith (2010), an initial encoding stage is inferred that delays the start of the second stage ("decision making") by an amount that is changed considerably (by 100 ms or more) by variations in stimulus noise; however, because the same factor also affects the decision stage, its influence with respect to those two stages is not selective.

Two stages with selective effects on both

Because two or more stages can be concatenated and treated as a single stage, we can limit consideration to processes consisting of two stages, without loss of generality. Consider a process that consists of stages **A** and **B** with durations T_A and T_B, so that the reaction time is $RT = T_A + T_B$. Assume that T_A and T_B are stochastically independent. We then have an *SIStage process* (a process consisting of sequential operations whose durations are stochastically independent; Roberts & Sternberg, 1993). Suppose two factors, F_j and G_k, each with two levels, $j = 1, 2$, and $k = 1, 2$, that influence the stage durations selectively, so that $T_A = T_A(F_j) = T_{Aj}$, $T_B = T_B(G_k) = T_{Bk}$, and $RT_{jk} = T_{Aj} + T_{Bk}$. Consider a 2×2 factorial experiment with the four resulting conditions, giving us RT_{11}, RT_{12}, RT_{21}, and RT_{22}. (Because an $m \times n$ experiment can be regarded as a concatenation of 2×2 experiments, we can do so without loss of generality.) Then, because convolution is associative and commutative, $RT_{11} + RT_{22}$ has the same distribution as $RT_{12} + RT_{21}$, namely, the convolution of the distributions of T_{A1}, T_{A2}, T_{B1}, and T_{B2}. Thus,

$$RT_{11} * RT_{22} = RT_{12} * RT_{21} \quad , \tag{1}$$

where * represents convolution. It follows that:

$$\kappa_{r11} + \kappa_{r22} = \kappa_{r12} + \kappa_{r21} \quad , \quad (r \geq 1), \tag{2}$$

where $\kappa_{rjk} = \kappa_r(RT_{jk})$ is the rth cumulant of RT_{jk} . Let us assume that RT distributions are "well-behaved", in the sense that cumulants of (at least) orders $r = 1, 2, 3$, and 4 exist.[2]

Eq. 2 results from three assumptions: (a) *Stages*, (b) *Stochastic Independence*, and (c) *Selective Influence*. To these, let us add a fourth assumption: (d) *Shape Invariance*: The RT_{jk} distributions differ by at most means and scale factors. Whereas differences among means influence only the means of the RT_{jk} distributions and influence none of the cumulants above the first, differences among scale factors influence all of the cumulants and central moments above the first. Now, if the distributions of two random variables, X_1 and X_2 have the same shape, with

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2. In what follows, two well-known properties of cumulants (κ_r) of order r are used: For stochastically independent random variables X and Y, $\kappa_r(X+Y) = \kappa_r(X * Y) = \kappa_r(X) + \kappa_r(Y)$ and $\kappa_r(CX) = C^r \kappa_r(X)$. Note that $\kappa_{rjk} = M_{rjk}$, for $1 \leq r \leq 3$ and $\kappa_{4jk} = M_{4jk} - 3M_{2jk}^2$, where the $\{M_{rjk}\}$ are the mean and r^{th} central moments of RT_{jk}. (See Kendall & Stuart, Volume 1, 1969, Ch. 3.) The quantity κ_4/κ_2^2 is a common measure of kurtosis, whose value is zero for the Gaussian distribution, and nonzero for other common distributions (Weisstein, 2014).

scale factor C, then

$$\kappa_r(X_2) \,/\, \kappa_r(X_1) = C^r \quad , \quad (r \geq 2). \tag{3}$$

Let the scale factor associated with RT_{jk} be $C_{jk} > 0$. It follows from Eq. (3) that

$$\kappa_{rjk} = C_{jk}^r \kappa_{r00} \quad , \quad (r \geq 2, \ \kappa_{r00} \neq 0). \tag{4}$$

where the $\{\kappa_{r00}\}$ are a set of constants, one for each r. With Assumption (d), Eqs. 2 and 4 then imply that:

$$C_{11}^r + C_{22}^r = C_{12}^r + C_{21}^r \ , \quad (r \geq 2). \tag{5}$$

Given that $\kappa_{200} \neq 0$ (nonzero variances) and $\kappa_{400} \neq 0$ (nonzero kurtosis values, which, among common distributions, excludes only the Gaussian) there are only three relations among the C_{jk} that satisfy Eq. (5)[3]:

(i) the C_{jk} are identical,
(ii) $C_{11} = C_{21}$ and $C_{22} = C_{12}$,
(iii) $C_{11} = C_{12}$ and $C_{22} = C_{21}$.

Given (i), only the mean RT and none of the higher cumulants can be influenced by either factor. Given (ii), factor F can influence only the mean. Given (iii), factor G can influence only the mean. Thus, for the four RT distributions to have the same shape, at least one of the two factors can cause no more than a *shift* (a change in mean only) of the RT distribution, a highly unlikely possibility.[4]

Two stages with a selective effect on one

This is sometimes assumed or concluded in applications of Ratcliff's (1978) diffusion model. Suppose the two-stage model, with one factor F_j ($j = 1, 2, , \ldots$) that influences just T_A, so that

$$RT_j = T_{Aj} + T_B \ , \quad (j \geq 1). \tag{6}$$

We then have

$$\kappa_{rj} = \alpha_{rj} + \beta_r \ , \quad (j \geq 1, r \geq 1), \tag{7}$$

where κ_{rj}, α_{rj}, and β_r are the rth cumulants of RT_j, T_{Aj}, and T_B, respectively. Equation (7) follows from assumptions (a), (b), and (c), above. Addition of the *Shape Invariance* assumption then requires

$$\kappa_{rj} = C_j^r \kappa_{r1} \quad , \quad (j \geq 2, r \geq 2), \tag{8}$$

where $C_j \neq 1$ is the scale factor that relates RT_j to RT_1. Combining Eqs. (7) and (8) and

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3. To prove this, use Eq. (4) with $r = 2$ and $r = 4$. For simplicity, let $C_{11} = a$, $C_{22} = b$, $C_{12} = c$, and $C_{21} = d$. Start with (A) $a^2 + b^2 = c^2 + d^2$ and (B) $a^4 + b^4 = c^4 + d^4$. Express the two sides of (A) and (B), respectively, in terms of $(a + b)^2$ and $(c + d)^2$, and of $(a + b)^4$ and $(c + d)^4$. Square the two sides of the equation derived from (A), and subtract from the equation derived from (B). This gives $ab = cd$, or $a/c = d/b = k$. Substituting in (A) gives $(k^2 - 1)(c^2 - b^2) = 0$. This implies either that $k = 1$, which means that $a = c$ and $b = d$; or that $b = c$, which requires $a = d$.

4. Effects of factors on mean RT are almost always associated with non-zero effects on other aspects of the distribution, including var(RT). Indeed, Wagenmakers & Brown (2007) have argued for a lawful regularity in the relation between mean and variance: they claim that the standard deviations of RT distributions increase linearly with their means.

rearranging, we have

$$\beta_r = \frac{(\alpha_{rj} - C_j^r \alpha_{r1})}{(C_j^r - 1)} \quad , \quad (j \geq 2 \, , \, r \geq 2). \tag{9}$$

Thus, either T_B is a constant ($\beta_r = 0, \;\; r \geq 2$) or its distribution (which is uniquely determined up to its mean by the $\{\beta_r\}, \;\; r \geq 2$) is restricted by properties of the distribution of T_A, and may vary with the level of the factor F_j that is assumed to influence only T_A, a contradiction.

Shape differences of RT distributions in flash detection

An example is provided by an experiment first reported by Backus & Sternberg (1988). It is called "Exp. 1" by Roberts and Sternberg (1993), who used the *Summation Test*, explained and applied in that paper, to show that the data are consistent with a SIStage model with selective influence.[5] Subjects responded by pulling a lever if, after a variable foreperiod, they detected a flash in one of four locations. A central cue at the start of each trial indicated the most likely location. On 25% of the trials ("catch" trials) there was no flash. The factors foreperiod (6 levels) and flash intensity (two levels) varied approximately randomly and independently from trial to trial.

For testing the SIStage model we used only the data from trials when the cue was valid and when the foreperiod was either 750 or 1150 ms. Six subjects provided these data, each of whom served for six one-hour test sessions after three hours of practice. The data considered here thus reflect four conditions that are, from shortest to longest mean RT, (short, bright; Sb), (long, bright; Lb), (short, dim; Sd), and (long, dim; Ld). As shown in Table 26.2 of Roberts & Sternberg (1993), the overall mean RT is 222 ms; the main effects of foreperiod and intensity on mean RT are 15 ms and 36 ms, respectively, and their interaction is close to zero. That the summation test is well satisfied is shown in Fig. 26.2A and 26.3A in Roberts & Sternberg (1993); Fig. 26.2A also shows the four distribution functions.

L-skewness and L-kurtosis

To reduce heterogeneity for the present analysis, the first of the test sessions was omitted. For each of the six subjects and each of the four conditions, the data were pooled over the remaining five test sessions. This resulted in 24 distributions of 80 observations each. For each distribution, estimates of the first four L-moments (Hosking, 1990, 1992, 2006; Hosking & Wallis, 1997; Royston, 1992), λ_1, λ_2, λ_3, and λ_4 and the derived measures of skewness, $\tau_3 = \lambda_3/\lambda_2$ ("L-skewness") and kurtosis, $\tau_4 = \lambda_4/\lambda_2$ ("L-kurtosis"), were calculated. Means and standard errors over the six subjects of estimates of λ_1, λ_2, τ_3, and τ_4 for the four conditions are provided in Table 1.[6]

Interpretations of $\hat{\tau}_3$ and $\hat{\tau}_4$ may be guided by the fact that they fall within the unit interval: $0 \leq \tau_3, \tau_4 \leq 1$, and that for the exponential distribution, $\tau_3 = .333$ and $\tau_4 = .167$, and for the

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5. Suppose the SIStage model together with selective influence of the manipulated factors, and a 2×2 factorial experiment with conditions 11, 12, 21, and 22 in a task for which these assumptions are valid. One prediction (Eq. 1) is that the convolutions of the distributions of RT_{11} with RT_{22} and of RT_{12} with RT_{21} should be equal (Ashby & Townsend, 1980). It is this prediction that is assessed, using numerical convolution, by the summation test, introduced with applications by Roberts & Sternberg, 1993. See their Section 26.2 and Appendix for a more formal proof and discussion.

6. L-moments are linear combinations of order statistics that are influenced less than conventional moments by extreme observations, and have other desirable properties (Hosking, 1992; Royston, 1992). Calculations were performed using the R-package "lmom".

Gaussian distribution, $\tau_3 = 0$ and $\tau_4 = .123$. The differences among the $\hat{\tau}_3$ values across conditions are striking: all six subjects show differences in the same direction for conditions Sb versus Lb and Sb versus Ld, and five of the six show a difference in the same direction for conditions Sb versus Sd.

Table 1

Means and Standard Errors of Estimates of Four Parameters

Condition	Sb	Lb	Sd	Ld
Foreperiod	Short	Long	Short	Long
Intensity	Bright	Bright	Dim	Dim
Measure				
$\hat{\lambda}_1$ (*ms*)	195.9 (4.5)	210.5 (3.5)	231.6 (3.2)	245.7 (3.9)
$\hat{\lambda}_2$ (*ms*)	9.46 (0.70)	13.27 (0.83)	12.71 (1.54)	15.63 (1.68)
$\hat{\tau}_3$	.067 (.035)	.257 (.029)	.183 (.023)	.210 (.031)
$\hat{\tau}_4$	.176 (.012)	.225 (.023)	.189 (.014)	.186 (.015)

Note. Rows = parameters; columns = conditions. $\hat{\lambda}_1$ = estimate of first L-moment; $\hat{\lambda}_2$ = estimate of second L-moment; $\hat{\tau}_3 = \hat{\lambda}_3/\hat{\lambda}_2$ = estimate of L-skewness; $\hat{\tau}_4 = \hat{\lambda}_4/\hat{\lambda}_2$ = estimate of L-kurtosis.

The mean difference between the means of the $\hat{\tau}_3$ values for conditions Lb, Sd, and Ld, and of the values for condition Sb, with $\pm SE$ is 0.15 ± 0.03; a t-test yields $p < .01$. (A similar comparison for the conventional measure of skewness, $\kappa_3/(\kappa_2{}^{3/2})$, yields a difference of $1.37 - 0.37 = 1.00 \pm 0.36$, and $p < .05$.) In an anova in which effects were compared to their interaction with subjects, the effects on $\hat{\tau}_3$ of foreperiod, intensity, and their interaction were all statistically significant (with p = .04, .02, and .02, respectively). In a similar anova for $\hat{\tau}_4$, the interaction of foreperiod with intensity was statistically significant, with $p = .02$.

How great are these effects on τ_3? One basis for comparison is the increase in skewness with the size of the positive set, n_{pos}, in "memory scanning", a shape difference that has been emphasized by several investigators (e.g., Hockley & Corballis, 1982; Hockley, 1984; McElree & Dosher, 1989). In simulations of the ex-Gaussian distribution based on Hockley's parameter estimates (Hockley, 1984, Fig. 4), the difference between the largest $\hat{\tau}_3$ (for $n_{pos} = 6$) and the smallest (for $n_{pos} = 3$) is .296 - .211 = .085 for positive responses and .302 - .267 = .035 for negative responses. In contrast, as shown in Table 1, the difference between the largest mean $\hat{\tau}_3$ (in condition Lb of the present experiment) and the smallest (in condition Sb) is .257 - .067 = .190.

Quantiles of normalized distributions

To investigate sources of the effects on $\hat{\tau}_3$ and $\hat{\tau}_4$, the RTs were transformed linearly to normalize the twenty-four distributions so that they had equal medians (217 ms) and interquartile ranges (25 ms), equal to the means across the distributions of their medians and interquartile ranges, respectively. For each normalized distribution a set of quantiles was estimated.[7] Let q_{pcs} be the quantile for a given proportion, p, condition, c, and subject, s. From the $\{q_{pcs}\}$, their means over conditions, $\{q_{p\bullet s}\}$, and the differences $Q_{pcs} = q_{pcs} - q_{p\bullet s}$ could be determined. It is

7. All quantile estimates used the Hyndman & Fan (1986) type 8 estimator.

the $\{Q_{pc\bullet}\}$, the means over subjects of these differences, called "adjusted quantiles", that are shown in Fig. 1.

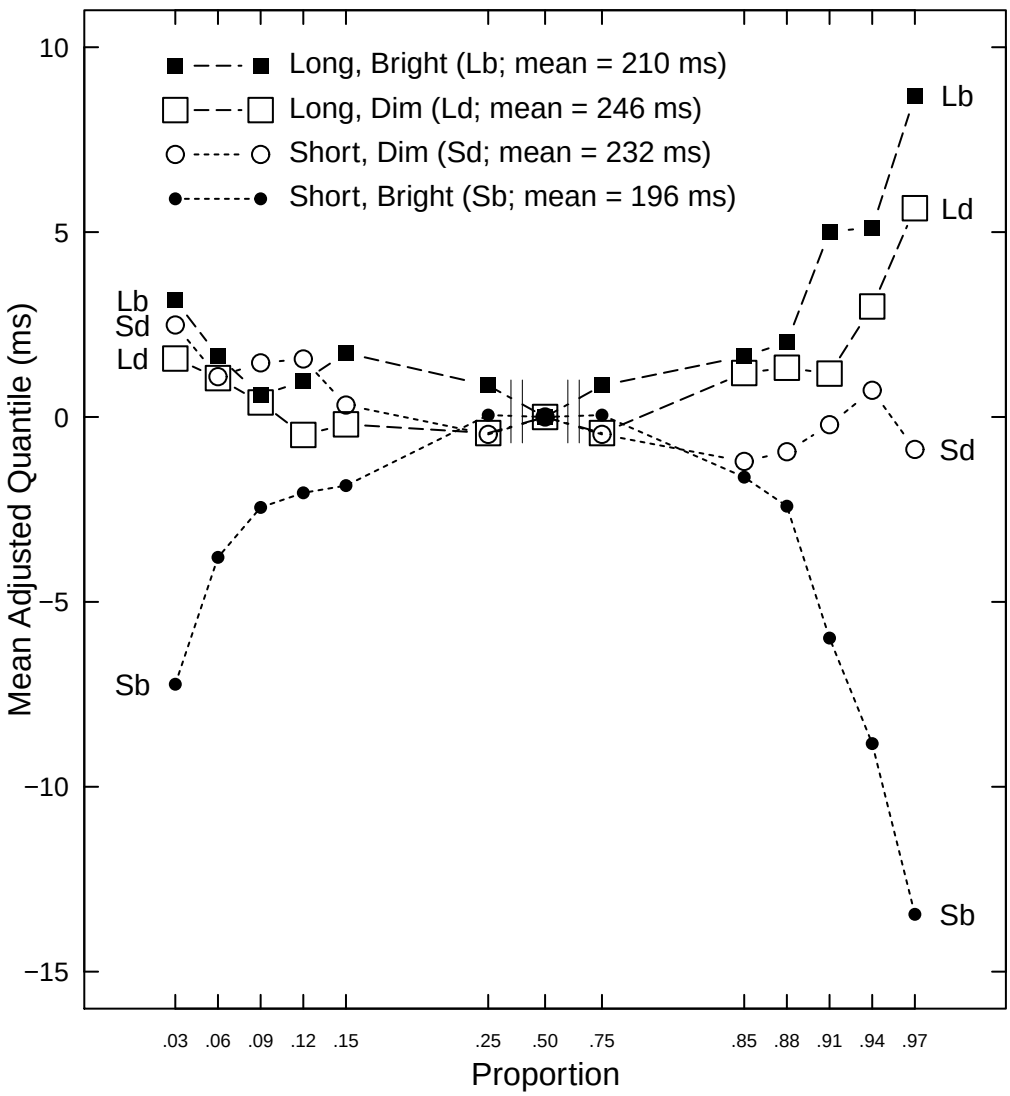

Figure 1. Data from four conditions in a detection experiment in which foreperiod and flash intensity were varied factorially: Means over six subjects of differences between the quantiles of normalized distributions in each condition and their means. To enable better visualization of the tails of the distributions, the x-axis is nonlinear, with breaks marked by vertical lines.

The interaction of the effects of foreperiod and intensity on τ_3 and τ_4 (striking, given that the effects of these factors on $\overline{RT}$ are additive) is also shown by their effects on the quantiles for both low and high tails: the effects of foreperiod on shape are substantially greater when the flash is bright than when it is dim. Separate anovas for low and high tails show that proportion, treated as a numeric variate, interacts significantly with condition (low tail: $p = .01$; high tail: $p = .003$)

and with the interaction of foreperiod with intensity (low tail: $p = .03$; high tail: $p = .02$). In an anova in which tail (low or high) is a factor, and proportion is measured outward from .5, the interaction of proportion, condition, and tail is highly significant ($p = .0001$), confirming the impression that the effects of condition on the high tail are greater than on the low tail. That a separation between the Sd and Lb conditions shows up only for the high tail hints at a qualitative difference between the two tails that could not be confirmed statistically.

It seems likely that these effects on the shapes of RT distributions reflect interesting properties of the underlying process; it remains to be determined what these properties are.

Shape invariance and the ex-Gaussian distribution

Given that the ex-Gaussian distribution has been fitted to numerous sets of RT data, it is interesting to ask about the conditions under which two different ex-Gaussian distributions have the same shape. For the exponential distribution with scale parameter δ, the first three cumulants are δ, δ^2, $2\delta^3$, and in general, $\kappa_r = (r-1)!\delta^r$. Those of the Gaussian distribution are μ, σ^2, 0, and for $r > 3$, $\kappa_r = 0$. The cumulants of the ex-Gaussian distribution are therefore the sums, $\delta + \mu$, $\delta^2 + \sigma^2$, $2\delta^3$, and, for $r > 3$, $(r-1)!\delta^r$. It is easy to show[8] from Eq. (3) that two different ex-Gaussian distributions have the same shape if and only if $\sigma_2/\sigma_1 = \delta_2/\delta_1$. Thus a minimum requirement for sameness of shape is that any factor that influences either δ or σ should also influence the other, and in the same direction. Yet in the Matzke & Wagenmakers (2009, Supplemental Materials) inventory of ex-Gaussian analyses, this weak requirement is met in only 31 (about 21%) of the 147 cases where effects on δ and σ and their directions were observed. Thus, to the extent that the ex-Gaussian distribution fit well, factor levels influenced the shapes of most of the RT distributions that were analyzed, violating shape invariance.

Conclusions

Given stages with variable and independent durations, and factors that influence more than the means of those durations selectively, the RT distributions in a factorial experiment are highly unlikely to have the same shape. Also, given variable and independent durations, and a factor that influences more than the mean of just one of those durations, the RT distributions for different levels of that factor are highly unlikely to have the same shape. Evidence from the fitting of the ex-Gaussian distribution to RT distributions has often revealed differences in shape. It follows that any theory that predicts shape invariance must be of limited generality. How well the distributions produced by an SIStage process can *approximate* shape invariance is a question for further research. (To the extent that the answer is "poorly", the observation of shape invariance in a particular case would constitute evidence against the SIStage model in that case.) In thinking about this issue it is important to consider the sensitivity of the standard tests for differences between distributions, and of the associated graphical displays. Acknowledgement of the existence of shape differences among RT distributions may lead to further understanding of the underlying processes.

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8. Let C be the scale factor that distinguishes the two distributions. From Eq. (3), $\kappa_{32}/\kappa_{31} = \delta_2{}^3/\delta_1{}^3 = C^3$, and $\kappa_{22}/\kappa_{21} = (\delta_2{}^2 + \sigma_2{}^2)/(\delta_1{}^2 + \sigma_1{}^2) = C^2$. The first of these implies $\delta_2/\delta_1 = C$. Combining this with the second gives $\sigma_2/\sigma_1 = \delta_2/\delta_1$, which is thus a necessary condition for sameness of shape. And because $\kappa_{r2}/\kappa_{r1} = \delta_2{}^r/\delta_1{}^r = C^r$ for $r > 2$, it is also a sufficient condition.

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